APPENDIX

It is desired to show that any cluster in the database that is closer to the vector V than cluster Cc must include points that are a distance between $(Dv + Cv)$ and $MAX\{0, (Dv-Cv)\}$ away from the indexing reference point. Herein we assume $(Dv - Cv) \geq 0$. The case in which $(Dv-Cv) < 0$ and is replaced by 0 proceeds by the same arguments.

Recalling that Cv is the distance between the input vector V and cluster Cc, consider the region of (N dimensional) space containing the point defined by vector V and all points that are a distance from 0 to Cv away from V. We call this a "ball" around point V, B_v. Recalling that Dv is the distance of point V from the indexing reference point, B_v contains a point closest to the indexing reference point and a distance $(Dv-Cv)$ or less from the indexing reference point. B_v also contains a point farthest from the indexing reference point at a distance $(Dv+Cv)$ from the indexing reference point. B_v may or may not contain the indexing reference point.

If Bv does not contain the indexing reference point, consider all points in space having a distance between $(Dv-Cv)$ and $(Dv+Cv)$ from the indexing reference point. We call this shell around the indexing reference point S_i. S_i contains all of the points of B_v. Any point outside of S_i lies at a distance less than Dv-Cv, or greater than Dv+Cv, from the indexing reference point. Also, since S_i contains all of B_v, no point outside of S_i will be in B_v and, therefore, no point outside of S_i will be closer to V than Cc. This means that any point less than a distance Dv-Cv or greater than Dv+Cv from the indexing reference point will be further from V than Cc.

If B_v contains the indexing reference point (that is, V is closer to the indexing reference point than it is to Cc), S_i becomes a sphere around the indexing reference point with radius Dv+Cv as the inner radius of the S_i shell collapses to zero. All points lying outside this sphere S_i also lie outside the sphere B_v so only points less than a distance Dv+Cv from the indexing reference point can be closer to V than Cc.